# LETTERS TO THE EDITOR

*Editor's Note:* Articles concerned with the settling of concentrated suspensions have a history of being very controversial. We have sometimes accepted such articles and encouraged discussions, as here, in Letters to the Editor to open the issues to the entire chemical engineering community. (Similar exchanges were published in Vol. 34, p. 1932 and Vol. 36, p. 633).

## To the Editor:

I would like to make some comments about the article titled "A Two-Dimensional Model for the Free-Settling Regime in Continuous Thickening" by B. Fitch (October 1990, Vol. 36, p. 1545).

The authors puts forward three concepts. The first concept is that a thickener can be viewed as a two-dimensional settling basin coupled with a one-dimensional consolidating vessel and that the "*area principle*" can be applied to it. He deduces an area principle by relating to the area of a rectangle, with the width of the basin and a Kynch characteristic as sides, as well as the solid volume flux and the propagation speed of concentration waves along the characteristic. He claims that the principle is generally valid. However, it is valid only under certain assumptions that are beyond the obvious assumption of Kynch regime. The main problem is that the author fails to establish a relationship between this "area principle" and the principle of constancy of the solid flux-density in a steady-state continuous thickener (consolidating vessel), and therefore the whole concept of a "two-dimensional free-settling regime" is superfluous. Details on two-dimensional settling basins are discussed by D'Avila et al. (1978).

The second concept in Fitch's article is that there exists an upper bound for the solid flux in a continuous thickener and that this upper bound is independent of the compressibility of the pulp. Even though this concept is correct, it was first discussed by Adorjan (1976) and therefore it is not new. It constitutes the essence of Adorjan's method of thickener design (Adorjan, 1976).

His third concept is that at present no theoretically-sound and empirically-confirmed procedure for designing thickeners exists. Following the idea that compressibility does not enter into area demands, he proposes to use "Tory plots" (free settling rates vs. reduced height in a settling column) to obtain the area demanded by a thickener by using a single batch experiment. Let's say that the "Tory plots" method is useful in obtaining the constitutive equation for suspension in Kynch regime and finding the area demands. It is obvious, however, that measuring the fall of the interface in a single batch experiment without simultaneously measuring other relevant variables such as the excess pore pressure distribution and the concentration distribution, both as functions of the height in the column and time, will never give sufficient information for a sound design procedure. See Been (1981), Been and Sills (1981b), and Tiller et al. (1990) for more sophisticated experimental procedures.

The author shows a strong mistrust in the use of mathematics for describing the sedimentation process. This is unfortunate considering his great reputation as a practicing engineer in the field of thickening and his long-standing position as a university professor and researcher. This can dissuade new research workers against adopting this approach that, I think, will lead to a general sedimentation theory. I agree with the author that at present no theoretically-sound and empirically-confirmed procedure for designing thickeners exists. He states that "currently used mathematical models have theoretical shortcomings and empirically the behavior of thickening suspensions deviates widely from that predicted by the models." It is important to be precise in this respect. It is possible today to apply a general approach to develop mathematical models for sedimentation processes. These models are general, physically-sound and mathematically-rigorous. They provide a framework to study the sedimentation of any suspension. Particular behaviors, such as compressibility and shortcircuiting, to mention only those cited by Fitch, can be introduced via constitutive equations that have to be defined theoretically and determined experimentally. The tools to do this exist, and the only thing missing so far is work by enthusiastic and well-trained researchers.

Fitch's view of thickening reminds me of the early approaches to thermodynamics, so well described in the "The Tragicomical History of Thermodynamics" (Truesdell, 1980) and the "Rational Thermodynamics" (Truesdell, 1984), which deal with the double standard that an author uses when he treats mechanics and when he uses thermodynamics. In our case, the same can happen when an author treats momentum, heat and mass transfer, and falls into the world of thickening.

To substantiate my statements I will have to write a few equations describing the theory of thickening, as I see it developing these days. I will show that equations developed intuitively and without mathematical consistency by Fitch can be obtained easily, stating assumptions and limitations from a set of field equations. I will also show some errors incurred by the author.

Let me, first, write the local balances of mass and momentum for the components of a flocculated mixture of a solid and a fluid. These equations have been accepted explicitly or implicitly by the principal research workers in the field of thickening, including Fitch himself. Then, I will apply them to the case of a two-dimensional settling basin coupled with a one-dimensional thickener.

### *Local mass and momentum balances at a steady state*

By making a macroscopic balance of mass and momentum, obtaining local balances in regions where the *field variables are continuous*, and making a dimensional analysis, we can write the following set of equations that represents a *dynamical sedimentation process*:

$$\frac{\partial \varphi}{\partial t} + \nabla \cdot f = 0, \quad (1)$$

$$\nabla \cdot \boldsymbol{q} = 0, \tag{2}$$

$$\nabla \cdot \boldsymbol{T}_s = -\rho_s \varphi \boldsymbol{g} - \boldsymbol{m}, \tag{3}$$

$$\nabla \cdot \boldsymbol{T}_f = -\rho_f (1-\varphi)\boldsymbol{g} + \boldsymbol{m}, \tag{4}$$

where $\varphi$ is the volume fraction of solids; $\boldsymbol{f} = \varphi \boldsymbol{v}_s$ is the solid flux density; $\boldsymbol{q} = \varphi \boldsymbol{v}_s + (1-\varphi)\boldsymbol{v}_f \equiv \boldsymbol{v}_s - (1-\varphi)\boldsymbol{u}$ is the volume-average velocity; $\boldsymbol{v}_s$ and $\boldsymbol{v}_f$ are the solid and fluid velocity; $\boldsymbol{u} = \boldsymbol{y}_s - \boldsymbol{v}_f$ is the relative solid-fluid velocity; $\boldsymbol{T}_s$ and $\boldsymbol{T}_f$ are the solid and fluid stresses; $\Delta\rho = \rho_s - \rho_f$ is the solid-fluid density difference; $\boldsymbol{g}$ is the gravity body force; and $\boldsymbol{m}$ is the solid-fluid interaction force.

These field variables are valid for any suspension, for which the particles are small with respect to the container and of the same size shape and density. To introduce the particular behavior of each material, constitutive equations should be postulated for the solid and fluid stresses and for the solid-fluid interaction force. These will describe hindered settling (or free settling as Fitch calls it) or flow through porous media, whichever is the case, and the compressibility of the sediment, respectively.

For the assumptions implied in Fitch's article, Eqs. 3 and 4 become:

$$\nabla \sigma_e = \Delta\rho\varphi \boldsymbol{g} - \frac{\alpha(\varphi)\boldsymbol{u}}{1-\varphi} \equiv \Delta\rho\varphi\boldsymbol{g} - \frac{\alpha(\varphi)}{(1-\varphi)^2}(\boldsymbol{v}_s - \boldsymbol{q}), \tag{5}$$

$$\nabla p_e = \frac{\alpha(\varphi)\boldsymbol{u}}{1-\varphi}, \tag{6}$$

where $\sigma_e$ and $p_e$ are the solid effective stress and the excess pore pressure, respectively, and $\alpha(\varphi)$ is the resistance coefficient for the solid-fluid relative flow.

*Two-Dimensional Settling Basin.* Consider a rectangular settling basin of width $W$ operating at steady state, where the feed enters homogeneously at $y=0$, $0 \le x \le L$. All the solid passes to the "thickener" at $x=0$, $0 \le y \le Y$, and the overflow fluid leaves through $y=Y$, $0 \le x \le L$. Since in the basin the suspension is in hindered settling (concentration less than the critical, that is, concentration below the one for which a solid network forms and where part of the weight of the solid is supported by the solid skeleton), $\sigma_e = 0$ and Eqs. 1, 2, 5 and 6 reduce to:

$$\frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} = 0, \tag{7}$$

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0, \tag{8}$$

$$v_{sx} = q_x - \frac{\Delta\rho\varphi(1-\varphi)^2 g}{\alpha(\varphi)}, \quad v_{sy} = q_y = v_{fy}, \tag{9}$$

$$\frac{\partial p_e}{\partial x} = -\Delta\rho\varphi g, \tag{10}$$

$$\frac{\partial p_e}{\partial y} = 0. \tag{11}$$

Equation 11 shows that the excess pore pressure $p_e$ is a function of the $x$ coordinate only, therefore from Eq. 10, we can conclude that $\varphi$ is also a function of the $x$ coordinate only in those regions where the variables ($p_e$ and $\varphi$) are continuous. This implies the existence of only two regions in the hindered settling domain: either $\varphi(x)$ or $\varphi = 0$ separated by a shock. Then, from Eqs. 7 and 9b, the following results:

$$\frac{\partial f_x}{\partial x} + \varphi\frac{\partial v_{sy}}{\partial y} = \frac{\partial f_x}{\partial x} + \varphi\frac{\partial q_y}{\partial y} = 0.$$

Using the continuity equation (Eq. 8) for the mixture, this equation becomes:

$$\frac{\partial f_x}{\partial x} - \varphi\frac{\partial q_x}{\partial x} = 0. \tag{12}$$

Multiplying Eq. 9a by $\varphi$ and defining the Kynch batch flux-density function $f_{bk}$ by:

$$f_{bk} = -\frac{\Delta\rho\varphi^2(1-\varphi)^2 g}{\alpha(\varphi)}, \tag{13}$$

we can write:

$$f_x = q_x\varphi + f_{bk}(\varphi). \tag{14}$$

Equation 14 gives the $x$ component of the solid flux density in terms of the $x$ component of the volume-average velocity and the Kynch batch flux-density function.

Substituting Eq. (14) into Eq. 12 yields:

$$q_x\frac{\partial \varphi}{\partial x} + + \frac{\partial f_{bk}(\varphi)}{\partial x} - \varphi\frac{\partial q_x}{\partial x} = 0,$$

$$q_x\frac{\partial \varphi}{\partial x} + + \frac{\partial f_{bk}(\varphi)}{\partial x} = 0. \tag{15}$$

Defining the transformation of coordinates $x = q_x t$:

$$\frac{\partial}{\partial x}(\cdot) = \frac{\partial}{\partial q_x t}(\cdot),$$

and if $q_x$ is independent of $x$ (then $q_y$ is independent of $y$):

$$q_x\frac{\partial}{\partial x}(\cdot) = \frac{\partial}{\partial t}(\cdot),$$

and Eq. 15 becomes:

$$\frac{\partial \varphi}{\partial t} + \frac{\partial f_{bk}(\varphi)}{\partial x} = 0. \tag{16}$$

Equation 16 represents the batch sedimentation of an ideal suspension in a settling column (one-dimensional model) (Bustos and Concha, 1988).

Therefore, the steady-state sedimentation in a two-dimensional settling basin may be represented by the Kynch equation for *batch sedimentation*, if the following two requirements are made: $x = q_x t$ and $q_x$ is independent of $x$.

On the basis of these general equations, I will analyze Fitch's article.

### *Area principle/thickener operation/cross-flow model*

In these sections, Fitch deduces an equation for the "area at which the characteristic planes reach the pulp-supernant interface" in a settling basin.

Following Fitch's idea, consider the volume of solids crossing a characteristic in batch settling. Let $Q_s$ be the volume rate of solids crossing a characteristic of concentration $\varphi_L$ at rate $v_L = f'_{bk}(\varphi_L)$, then:

$$Q_s = S(v_s(\varphi_L) + v_L(\varphi_L))\varphi_L, \tag{17}$$

where $S$ is the cross-sectional area of the settling column. Since the characteristic is a straight line $v_L = x_L/t_L$,

$$Q_s = S\left(v_s(\varphi_L) + \frac{x_L}{t_L}\right)\varphi_L. \tag{18}$$

Using the transformation $x_L = q_x t_L$ to come back to a two-dimensional model, Eq. 18 becomes:

$$Q_s = S(f_{bk}(\varphi_L) + q_x\varphi_L),$$

then, using Eq. 14:

$$S = \frac{Q_s}{f_x(\varphi_L)} . \qquad (19)$$

where $x_L$ and $t_L$ are the coordinates of the point where the characteristic reaches the water-suspension interface.

Since $f_x$ and $q_x$ are independent of $x$, see Eq. 12, and at the discharge of the thickener, $f_x = f_D$, $q_x = q_D$, and $f_D = q_D \varphi_D$, from Eq. 19 we have:

$$S = \frac{Q_s}{q_D \varphi_D} , \qquad (20)$$

where $Q_s$ is the volume flow rate of the solid, and $q_D$ and $\varphi_D$ are the volume-average velocity and the concentration at the discharge. Equation 20 relates the thickener area to the "area principle."

Equation 5 in Fitch's article is:

$$A_{ch} = \frac{Q_s}{v_L} ,$$

$$= \frac{Q_s}{x_L/t_L} .$$

Using the transformation $x_L = q_x t_L$, the following result is obtained:

$$A_{ch} = \frac{Q_s}{q_x} . \qquad (21)$$

What is the relationship between $A_{ch}$ and the thickener area? Fitch does not give it, and he does not use this concept further in his article. Then, why introduce the concept of a cross-flow model?

The author's assertion in his Eq. 5 that the area principle is valid "regardless of magnitude and direction of the flow vectors" is not true as can be seen from the previous analysis. It is required that the vertical component of the solid flux density and the volume-average velocity be independent of the vertical coordinate. Fitch's expression "regardless of the divergence of the flow vectors" is nonsense, since, as we saw at the beginning for the steady-state sedimentation, the flux density and the volume-average velocity are solenoidal, that is, the divergence of the flow vectors ($\boldsymbol{q}$ and $\boldsymbol{f}$) is always zero (see Eqs. 7 and 8). In the section on "Cross-Flow Model," the author repeats the same argument adding the "inclination" of the suspension. I assume that he refers to the inclination of the bottom of the settling basin. If this is the case, the situation is more complex because on inclined surfaces different phenomena occur (Acrivos and Herbolzheimer, 1979; Amberg and Dahlkild, 1987).

*Unit Area by Extrapolation.* In this section, Fitch gives a contradictory definition of the "free settling rate." First and correctly, it is defined as the "settling velocity in the absence of (effective solid) pressure gradient." But, later in the Notation section, the author adds that it is the "solid velocity relative to a system in which the total flux is zero." The first definition includes batch and continuous settling (as it should be, since free settling exists in both regimes), but the second definition restricts the free settling rate to batch settling. The problem arises by calling *free settling* to the product of the porosity and relative solid-fluid velocity (the accepted name of this term is *drift velocity*). The problem vanishes by calling free (I call it hindered) settling velocity $v_s(\varphi)$ to the velocity of the solid component (batch or continuous) in the absence of the effective solid pressure gradient. Then, the relationships among the free settling velocity $v_{sx}$, the volume average velocity $q_x$, and the relative solid-fluid velocity $u_x$ are straightforward:

$$q_x = v_{sx} - (1-\varphi) u_x \qquad (22)$$

For batch sedimentation $q_x = 0$, so that Eq. 22 reduces to $v_{sx} = (1-\varphi) u_x$, as the second definition of Fitch.

With these reservations and assuming that the dynamic liquid pressure used by Fitch corresponds to the excess pore pressure, the equation for $u^*$ in Fitch's article is correct, except for the missing term $c$:

$$u^* = Kg(\rho_s - \rho_f)c/\mu .$$

### *Effect of compression*

Fitch's equation for the height of the compression zone at the end of a batch test is wrong. It should be:

$$h = -\frac{1}{\Delta\rho g} \int_{\varphi_D}^{\varphi_c} \frac{\sigma_e'(\varphi)}{\varphi} \, d\varphi, \qquad (23)$$

where $\varphi_c$ is the critical concentration. The integration must be done from the discharge concentration $\varphi_D$ to the critical concentration $\varphi_c$. In Fitch's Notation, the previous equations become:

Min. Compression Depth =

$$\int_{c_c}^{c_u} \left(\frac{dx}{dc}\right)^* \frac{S}{S-G} \, dc,$$

where the notation is given in his article. The critical concentration $c_c$ may be replaced by the initial concentration $c_o$.

It is important to point out at this point that Fitch uses the term *critical* concentration in the sense of *limiting* concentration. These terms have been widely accepted with their original meaning.

## Notation

$f$ = solid flux density vector
$f_x$ = vertical component of the solid flux density vector
$f_y$ = horizontal component of the solid flux density vector
$f_{bk}$ = Kynch batch flux density function
$f_D$ = solid flux density at the underflow
$\boldsymbol{g}$ = gravity body force
$g$ = magnitude of the gravity force
$\boldsymbol{m}$ = solid-fluid interaction force
$P_e$ = excess pore pressure
$\boldsymbol{q}$ = volume average velocity vector
$q_D$ = volume average velocity at the underflow
$q_x$ = vertical component of the volume-average velocity vector
$q_y$ = horizontal component of the volume-average velocity vector
$t$ = time
$\boldsymbol{u}$ = relative solid-fluid velocity vector
$u$ = magnitude of the solid-fluid velocity vector
$v_f$ = fluid component velocity vector
$v_s$ = solid component velocity vector
$v_{sx}$ = vertical component of the solid velocity
$v_{sy}$ = horizontal component of the solid velocity
$v_{fx}$ = vertical component of the fluid velocity
$v_{fy}$ = horizontal component of the fluid velocity
$x$ = vertical coordinate
$y$ = horizontal coordinate
$Q_s$ = solid volume flow rate
$S$ = cross-sectional area of the thickener
$T_f$ = fluid stress tensor
$T_s$ = solid stress tensor
$\alpha(\varphi)$ = coefficient of resistance for the solid-fluid relative flow
$\rho_s$ = solid density
$\rho_f$ = fluid density
$\Delta\rho$ = solid-fluid density difference
$\varphi$ = volume fraction of solids

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## Reply:

Professor Concha makes one valid point in his comments. The variable $c$ was inadvertently omitted from the equation for $u^*$ on p. 1548. This should be obvious from its derivation given directly above. The equation is not used in the derivations that follow. It was introduced to establish that Darcy's law is implicit in $u^*$ and that extrapolation on the basis of $u^*$ is completely equivalent mathematically to extrapolation on the basis of $K$. Although some published articles would imply the contrary, substitution of variables does not change a mathematical model, but merely gives an alternative way of expressing it. The model when expressed in terms of $u^*$ is no less "Darcian" than when $K$ is left as an explicit variable in its expression.

The remainder of Concha's comments are based on misconceptions and misrepresentations of the literature, and misinterpretations of the subject article. They will be responded to item by item.

*Paragraph 1.* Anyone who had ever observed a thickener would be aware that the feed zone was not one-dimensional. Obviously neither is the compression zone, since underflow is taken from a single central point (Turner and Glasser, 1976). However, while a model that is hydraulically one-dimensional may be of some approximate relevance for the compression zone, it will clearly not be so for the feed zone. A two-dimensional model for the feed zone was, therefore, investigated. The two-dimensional model applies only to the free-settling feed zone, and not at all to the subjacent compression zone (as indicated by its title). With respect to the feed zone model, no assumption at all is made about flows in the compression zone, except that the boundary between the feed zone and the subjacent compression zone extends over the entire thickener area.

The area principle is deduced from a material balance around a differential element, bounded on the sides and above and below by flow lines. In the derivation, no restrictions are placed on the inclination of the flow vectors or on their divergence. In fact, the derivations explicitly show inclination of the flow lines (Figure 2 of my article). Therefore, there is no such restriction on the integrated equation that expresses the area principle. Thus, quite contrary to Concha's contention it is generally valid for a two-dimensional flow in the free-settling regime, regardless of the inclinations or divergences of the flow vectors. It is valid, however, only for the two-dimensional free-settling regime postulated. That is all that is claimed for it.

Concha writes, "The main problem is that the author fails to establish a relationship between this 'Area principle' and the principle of constancy of the solids flux-density in a steady-state continuous thickener (consolidating vessel), and therefore the whole concept of a 'two-dimensional settling regime' is superfluous." This is a complete logical nonsequitur and does not make sense. The area principle has nothing to do with the principle of constancy of "flux density" assumed in the subjacent compression zone. It is not clear what he means here, but apparently it is that in a one-dimensional model the flux is independent of $x$. This would be true only if the column is of constant area, and it does not follow at all that local flux is independent of $y$ in the superjacent feed zone. All that is required by steady-state operation is that the concentration at any point remain constant, and the total solids and liquid flows past any level be constant. This is as true for the two-dimensional feed zone as it is for the subjacent compression zone. No relationship of the sort called for by Concha has been shown as necessary, and indeed none exists.

The flow patterns in the compression zone will not actually be one-dimensional, which may affect the location of the boundary between the two zones. This will, in turn, affect the flow patterns in the feed zone, but will not affect the validity of the area principle, which is valid for the two-dimensional feed zone regardless of its flow patterns.

Details on two-dimensional settling basins can be found throughout the literature, particularly in the sanitary engineering field, going back at least as far as Hazen (1904).

*Paragraph 2.* Concha agrees that there exists an upper bound for the solids flux in a continuous thickener and that this upper bound is independent of the compressibility of the pulp. Concha writes that this concept was expressed first by Adorjan (1976). This indicates a lack of familiarity with the literature. The concepts are discussed in Fitch (1966) and form the basis for the thickener design method given there. They are also discussed in subsequent articles (1972, 1975, 1979, 1991). The design method of Adorjan is simply a mathematical variant of the original one. It derives one of the "other equations" deduced "de novo in the literature" mentioned in the article. As noted, it starts from the same basic assumptions as the earlier models and solves the same problem, namely the concentration profile through the compression zone of a continuous thickener. It uses a somewhat different mathematical route, but the derived equations are mathematically equivalent and can be interderived by appropriate substitution of variables. (For this not to be true, there would have to be a mathematical error in one of the derivations.)

*Paragraph 3.* Concha agrees that there is as yet no theoretically-sound and empirically-confirmed procedure for designing thickeners. From a logical standpoint, a sound procedure is one deduced from valid assumptions. As noted in Fitch (1979), there is as yet no reason to accept that the existing mathematical models for compression are based on valid assumptions. There is also much reason to suspect that they are not.

The variables that Concha says are relevant and necessary for the design of a thickener are, in fact, not at all necessary. All that is needed is equations for $u^*$ and $(dx/dc)^*$ as functions of $c$. (See Eq. 16 in the article, or the last equation in Concha's letter.) The variables Concha lists are implicit in these functions, which are dimensional groups quite analogous to dimensionless number groups. The variables $K$, $\mu$, $\rho_s$, $\rho_f$, and $d\sigma/dc$ do not have to be evaluated individually, if the functions are determined experimentally. Methods for determining them directly from simple and "unsophisticated" tests are described in Fitch (1966) and in later articles. It would seem naive indeed to make more complicated, "sophisticated" experiments to determine them.

The extrapolation method is not dependent on Tory plots. As noted, any extrapolation procedure could be used. Unit areas are then determined by Kynch methods or their mathematical equivalents from the extrapolated settling plots. Tory plots simply offer a convenient method for the extrapolation, since the $K_1$ range is empirically linear. The principal merit of Tory plots, however, is that they facilitate comparison of batch settling tests made under different initial conditions.

*Paragraph 4.* It is a complete misinterpretation of the article to infer from it that I mistrust mathematics. What I mistrust is not mathematics, but mathematicians. I have been questioning the assumptions they have been making in deriving their models. Every mathematically-derived model starts from a set of assumptions and equations that purport to model the local behavior of a prototype system at point in space and time. They are usually partial differential equations. When solved subject to appropriate boundary conditions, the results predict macroscopic behavior for the modeled systems. The mathematical derivations show how the system would behave if the axiom equations were valid or relevant. They can do this with spectacular power and elegance, but that's all. They prove nothing about validity of the axiom equations themselves. That has to be established empirically. If the axiom equations do not accurately represent the physical mechanisms operating, the results of the mathematical derivations, however elegant, may not describe what actually happens. In the present case, for example, derivations from a one-dimensional continuity equation cannot describe what actually happens in a two-dimensional region.

As computer scientists say, "garbage in, garbage out."

*Paragraph 5.* This comment is gratuitous, and its implication is false. My models, like just about everyone else's, are derived from mass and momentum balances.

*Paragraph 6.* Another comment with an unwarranted implication! As those who fully understood the literature could attest, my publications contain no examples of equations that are "developed intuitively and without mathematical consistency." In some instances, they are developed with the aid of logical deductions, which are just as valid and rigorous as purely mathematical ones, and are more fundamental. Logical rigor insures mathematical consistency, since as mathematical philosophers recognize, mathematics is just a specialized form of symbolic logic. However, logical deductions may seem "intuitive" to those with no understanding of logic. A direct comparison of a narrowly mathematical approach and a logical one is given in Fitch (1991).

*Item 7.* In paragraph 7 and succeeding sections, Concha develops his version of a two-dimensional model. While his derivations are doubtlessly mathematically impeccable, they are also irrelevant and constitute a clear-cut example of "garbage in, garbage out."

Figure 1 shows a physical interpretation of Concha's mathematical model. New feed enters a column or tower of free-settling suspension, bounded by shocks. To satisfy steady-state, the concentration existing at any level in the column must be such that the upward propagation rate of its Kynch characteristic, $v_K$, is precisely offset by the downward $v_u$ velocity of the system as a whole. Since $v_u = Q/A$, the concentration at any level must be such that:

$$A = Q/v_K \quad (1)$$

The area at the top of the section must be such as to make the feed concentration limiting. The area at the bottom must be

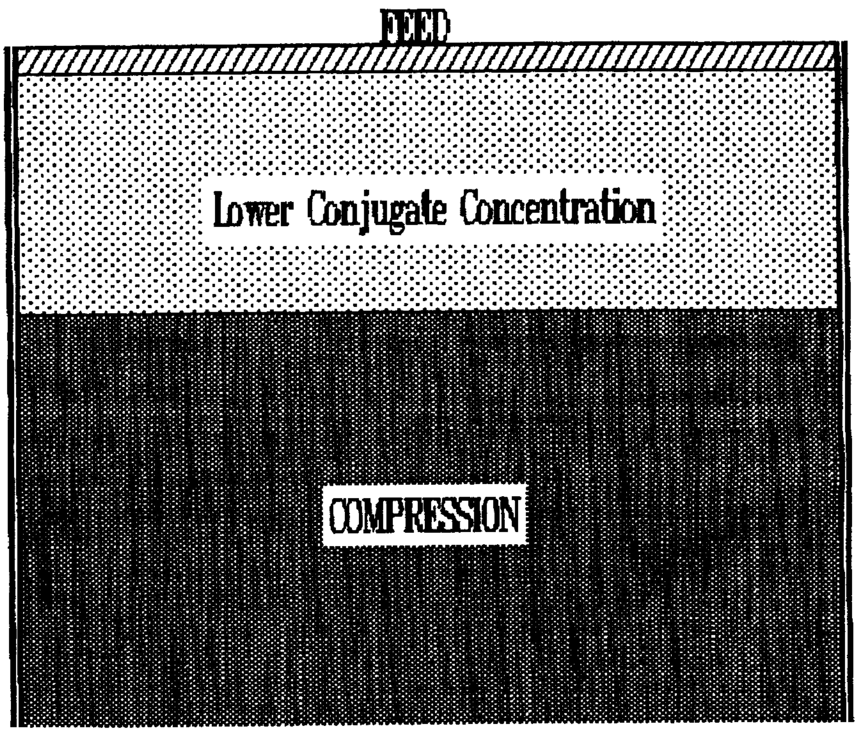

**Figure 1. Physical interpretation of Concha's mathematical model.**

such as to make the critical concentration for the thickener limiting. Therefore, the area at any level must increase as depth increases. The bounded column does not have to be conical, as long as its area increases with depth, but it may be. (Note that there is an inconsistency of Notation in the article not noted by Concha. In the text $Q$ is defined as total or system flow . In the Notation section, it is defined as solids flow. The former definition is the one used in the mathematical derivations.)

There are several problems with Concha's model. We will first show a fatal defect. If such a column were established, as by bounding it with an impermeable membrane, the pressure in the column at any level would exceed that in the surrounding clear liquid. If the bounding membrane were then removed, the column would erupt into its surroundings. It would slump to some other flow pattern. The idea that the column could be bounded by shocks was dragged in to avoid the completely ridiculous conclusion that the concentration at any depth in the thickener would be constant over its entire area, and thus that the concentration at the overflow would equal that in the feed. No mechanism warranting assumption of such shocks was derived or exists.

While Concha's model, if considered to be a fluid dynamic derivation of the flow patterns in a two-dimensional region, is scientifically unsound, his derivations do constitute a confirming instance of the area principle.

According to the area principle, every two-dimensional flow pattern will yield the same area demands for free settling. Concha's model defines a flow pattern, which, although totally unrealistic, is a conceptual two-dimensional one. It should yield the same area demands as were found in the article. Note that it does. In Eq. 1 above, $A$ is the area covered by the characteristic at that level and therefore corresponds exactly to the $A_{ch}$ of the article. Thus, Eq. 1 corresponds precisely with Eq. 5 of the article.

The purely hypothetical "ideal" cross-flow model given in the article did not pretend to be a valid model for the flow patterns in an actual basin. It, too, was simply another conceptual model, relevant to actual flow patterns only through the area principle. In fact, it was pointed out that a precise flow model would be extraordinarily difficult to write, but because of the area principle, was completely unnecessary for thickener design.

No further analysis is necessary to establish that the derivations in the criticism are not relevant to what is treated in the article. In particular, they cannot be invoked to refute any of its conclusions.

*Item 8.* Since Concha's mathematical model is scientifically unsound, any further derivations made from it are mathematical garbage, from a scientific standpoint. We will, however, comment on misconceptions and confusions in his section on "Two-Dimensional Settling Basin."

The hypothetical cross-flow basin is assumed to be fed at $y=0$ and $0<=x<=L$. This establishes that the concentration at $y=0$ is identically the feed concentration through the entire depth of the basin. Since under his model concentration it is a function of $x$ only, this entails that the concentration in the settling column is everywhere the same as that in the feed, and many of his subsequent derivations assume this restriction. It is possible for the solids to collapse directly from feed concentration into compression, but only if the feed concentration is the limiting one. It is not true in general, and Concha's model is not consistent with a cross-flow basin.

Concha writes, "Therefore, the steady-state sedimentation on a two-dimensional settling basin may be represented by the Kynch equation for batch sedimentation, if the following two requirements are met: $x=q_x t$ and $q_x$ is independent of $x$." One has to guess at what point and under what conditions he means $x$ and $t$ to apply, but presumably he refers to the $x$ coordinate of the liquid-suspension interface as a function of time. $q_x$ is by definition the $x$ component of $q$, earlier defined as $\phi v_s+(1-\phi)v_f$, which is the system velocity. In a batch sedimentation, the system velocity and thus also its $x$ component are both 0, unless the test is performed in an elevator, in which case it is identical to the velocity of the elevator. In batch test coordinates, however, $q_x$ is identically zero. Therefore, the equation $x=q_x t$ implies that the interface does not change with time, and thus the solids are not settling at all. Under these conditions, $q_x$ will indeed be independent of $x$, since it is everywhere zero. In a continuous operation, if the interface subsides with respect to system coordinates at the same rate as $q_x$, then again the solids cannot be settling. Obviously, these derivations are doubly divorced from all physical or scientific reality.

*Item 9.* The same is true for the derivations in the section "The Area Principle/Thickener Operation/Cross-Flow Model."

Concha's Eq. 19 could have been written for his model directly by inspection without mathematical meandering, if the logical relationships had been recognized. The definition of $f_x$ is $Q_s/S$. $f_x$ and $q_x$ are not independent of $x$, unless the column has a constant $S$.

Equation 20 gives the area $S$ in the column at the underflow level (assuming underflow is removed uniformly over the entire area), but not at any other level unless $S$ is constant.

The equation following Eq. 20 does not correspond to Eq. 5 of the article, in that Eq. 5 contains $Q$, not $Q_s$ (see note given earlier).

Concha asks what relationship exists between $A_{ch}$ and thickener area $A$. This was pretty much covered in the section "Thickener Operation" of the paper. $A_{ch}$ is the area that would be necessary for the characteristic to emerge at the pulp-supernatant interface and disappear. It is independent of the thickener area. If $A_{ch}$ is greater than $A$, the characteristic simply does not emerge.

Concha next returns to the question of inclination and divergence of flow vectors. It is not clear exactly what problem he sees, but it can be pointed out that if flow vectors could not diverge, flow could not spread out radially in a circular thickener. Continuity equations (Eqs. 7 and 8) do not bar divergence of flow lines, rather they constrain fluxes. As to inclination of the flow vectors, that has been treated above. If the bottom of the basin inclines, then the flow vectors would also have to be inclined. In this case, the fluid dynamic situation is indeed complex, but the area principle makes it unnecessary to deal with this complexity for determining basin area demands.

*Item 10.* Concha writes, "in this section (Unit Area by Extrapolation), Fitch gives a contradictory definition of the 'free-settling rate'." He seems totally confused. In the first place, the definition of "free-settling" apparently attributed to the section by Concha is not given there or anywhere else in the article. Free-settling rate $u^*$ is defined (after correcting the typo) by the equation:

$$u^* = Kg(\rho_s - \rho_f)c/\mu$$

which is fully consistent with the definition given in the Notation. Settling rates were defined as: "First, it should be noted again that settling rates are here defined as particle velocities in a coordinate system, in which the total volumetric flux (solids + liquid) is zero, i.e., in batch settling coordinates." Free settling rate or $u^*$ is thus the solids velocity in batch coordinates in the absence of compression gradients. The fact that free-settling rate is defined as the value that would be obtained in batch settling does not in any way restrict the use of this value in equations relating to other than batch settling, as indicated by Concha. The contradiction noted by Concha is thus between the definition given for "free settling" in the article and his own private one. There is not a contradiction within the article.

Concha repeatedly quibbles about terminology. There is no lexical authority defining the meaning of such terms as "free settling." They mean what they are used to mean. This is given by a definition in the article, or by customary usage in the art. Free settling was not only defined in the article, but has been used in the thickening art for at least three quarters of a century with the same meaning. The term "hindered settling" was not used. In the ore-dressing art, "free-settling" has been used for just as long to denote settling of single particles, and settling in suspensions has been termed "hindered settling." It would take a certain intellectual bigotry to hold that either one was the "correct" meaning. The same principle applies to other terms Concha questions.

*Item 11.* If $G$ has everywhere the value 0, the final equation in Concha's criticism becomes equivalent to both the equation given for minimum compression depth in the article and Concha's corresponding equation. They differ only in the lower limit of integration. However, if $c_0$ is not higher than $c_c$, the differential $(dx/dc)^*$ will have the value zero between $c_0$ and $c_c$. One will get the same answer no matter which is used for the lower limit. It is not at all clear why Concha deems the equation wrong.

It has been shown that Concha's criticisms are based on misconceptions, misrepresentations, and misinterpretations. They are totally refutable. His thickener model is fluodynamically unsound, and his derivations are irrelevant to what is presented in the article. He has not disclosed any errors other than one typographical one.

## Notation

Concha uses a different notation in his critique than was used in the article. This leads to some duplication and confusion.

- $A$ = cross-sectional area ($S$ in Concha's Notation)
- $A_{ch}$ = area covered by characteristic in steady-state thickening
- $c$ = volume fraction solids ($\phi$ in Concha's Notation)
- $c_0$ = initial concentration
- $c_1$ = upper conjugate concentration
- $G$ = settling flux
- $K$ = Darcian permeability
- $L$ = height of compression interface
- $Q$ = system flow
- $Q_s$ = solids flow
- $q$ = system velocity
- $q_x$ = $X$ component of system velocity
- $S$ = free-settling flux, $cu^*$
- $u^*$ = settling rate in the absence of pressure gradient, with respect to batch settling coordinates
- $v_K$ = propagation rate of Kynch characteristic with respect to system
- $v_l$ = velocity of liquid
- $v_s$ = velocity of solids
- $v_u$ = velocity of system due to underflow withdrawal
- $(dx/dc)^*$ = inverse of concentration gradient in a batch test after subsidence is complete
- $x$ = distance in direction of settling
- $y$ = distance perpendicular to $x$
- $f$ = density of fluid phase
- $s$ = density of solid phase
- $\mu$ = viscosity
- $\sigma$ = solids pressure
- $\phi$ = volume fraction solids (Concha's Notation)

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## To the Editor:

Fitch's analysis of the free-settling regime in continuous thickening (October 1990, Vol. 36, p. 1545) appears on first reading to be an extension of his classical 1956 analysis of the clarification process. As in this earlier analysis, two-dimensional flow in the free-settling concentration range is considered here. Unlike the earlier analysis, however, spread of a concentration gradient into the two-dimensional region is allowed. This leads to the development of a modified design procedure that occupies the remainder of the article.

It is difficult, however, to follow the author's argument, and to see if the analysis is actually two-dimensional. The observations made under "Relationships between Batch and Continuous Thickening" are essentially the same as those arising from the one-dimensional analysis of Font (1990), I discussed earlier (1990).

Under normal operation, the bulk of the incoming liquid leaves as overflow, while the remainder, together with (ideally) all the solids, leaves with the thickened underflow. Under the laminar flow assumption inherent in the analysis under discussion, the underflow liquid will be a lower layer (in a rectangular thickener) at the inlet weir, and the upper portion of the flow will be destined for the overflow. The boundary between these two liquid streams will be (in a two-dimensional elevation) the "lowest overflow streamline." The primary requirement for complete clarification (as distinct from thickening) is that, for a vertical element of overflow liquid pass-

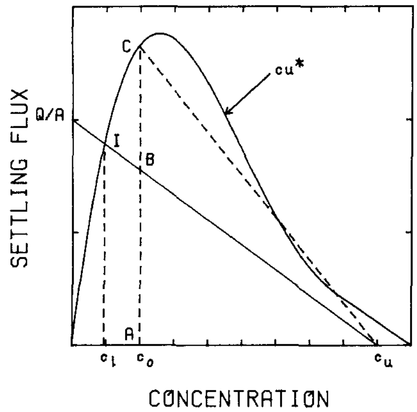


**Figure 1. Flux plot.**

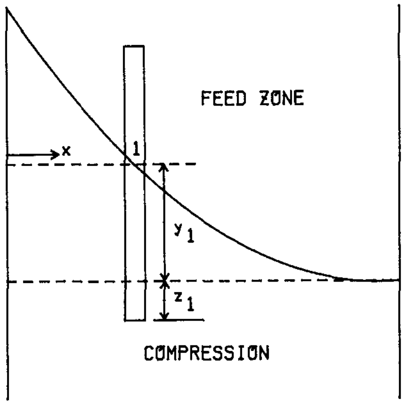


**Figure 2. Calculation of horizontal profile at a given level.**

ing from inlet to outlet, solids entering at the top are able to settle to the bottom, crossing the lowest overflow steamline, before the exit is reached.

The criterion for complete removal of solids of given settling velocity is that the overflow liquid flux not exceed the settling velocity. This was originally determined by Hazen (1904) on the basis of a highly idealized flow pattern. Fitch (1956) showed that the criterion also applied under much less restrictive assumptions about the flow.

If the operating conditions are such that the full clarification capacity of the thickener is used, solids leave the overflow stream uniformly over the whole horizontal cross-section. If the clarification capacity is not fully utilized, however, solids leave the overflow stream over only part of the area; but, as the solids progress downward after leaving the overflow stream, it is to be expected that they will spread out over the remainder of the cross-section. It appears that it is this spreading process that the author is addressing in his two-dimensional analysis of the free-settling regime, rather than the flow in the overflow stream.

The situation can be illustrated on the standard Yoshioka flux plot used for analyzing the thickening process, as shown in Figure 1 here and used by the author (Figure 13). On the assumption of uniform downward plug flow of liquid and solid in the thickening process, there is a straight operating line relating solids settling flux $cu$ and concentration $c$. The slope is $-v_u$, the abscissa intercept $c_u$, and ordinate intercept solids throughput flux, $Q_s/A$. ($Q$ is shown as solids flow under the Notation in the Fitch article, but it appears that this should be $Q_s$, as used in Eq. 6. $Q$ is total volumetric flow rate, as implied through Figure 2 and the derivation of Eq. 5.) The operating line has a single intercept with the no-compression settling line, $cu^*$ vs. $c$, at concentration $c_i$. For fixed $Q_s/A$ and $c_u$, the feed concentration $c_0$ must be greater than or equal to $c_i$ for the clarification capacity not to be exceeded. If $c_0$ satisfies this criterion, as in the figure, the clarification capacity is used only partly, and solids leave the overflow stream over fraction $AB/AC$ of the area.

As I asserted previously (1977, 1985, 1988), while free-settling conditions prevail (that is, while the solids are sufficiently far from the sediment not to be retarded by it), the solids stream can decrease only in concentration, approaching $c_i$, during the spreading process. The forces acting under free-settling under these conditions are such that the solids are accelerated and decrease in concentration. $c_i$ is the only concentration for which the no-compression settling flux matches the flux demanded by the material balance.

It appears that the primary aim of the author is to show that, contrary to the above, concentrations higher than $c_0$ can form in the free-settling zone of a continuous thickener. For batch thickening, starting at $c_0$ as shown in Figure 1, the spread of concentration gradients into the free-settling zone is predicted by Kynch kinematics [based on the $u = u^* = u^*(c)$ assumption], since $c_0$ is in region 2 or 3. (This assumes that there is a maximum concentration at or close to $c_u$). On the other hand, $c_i$ is in region 1, and so no gradient formation is predicted if the solids spread over the cross-section and thin to this concentration.

In Figure 3 of the Fitch article, the feed is treated, it seems, as a point source at the top lefthand corner. The solids spread over the cross-section, as they move downwards. Essential questions to be asked are: What are the assumptions made about the liquid flow? Where is the lowest overflow streamline assumed to lie? These questions are valid, since above the lowest overflow streamline the liquid flow is predominantly horizontal toward the overflow point, while below the lowest overflow streamline it is predominantly vertical toward the underflow offtake.

The author's consideration of a vertical column of suspension moving from left to right suggests a column of overflow liquid, from which solids settle, as in the original Hazen plug-flow clarification model. However, the author also assumes a uniform downward velocity $v_u$ throughout the feed zone, which suggests flow below the lowest overflow streamline. The two are incompatible.

It seems that the clarification process, in fact, is being neglected: the lowest overflow streamline is assumed to pass between feed and overflow points close to the top surface. Thus, the thickener operates well below the clarification limit; the solids pass below the lowest overflow streamline close to the inlet point; essentially the whole of the feed zone is taken to be occupied by liquid and solids that will pass out with the underflow.

This flow pattern, however, is not possible; it involves discontinuities at the side boundaries that cannot be avoided by any reasonable assumption. As a column progresses to the right, liquid and solids must progressively leave it, as they are distributed over the cross-section. The envisioned passage of effectively batch settling columns from one side to the other means that all the underflow liquid and solids are deposited against the far side of the thickener. (Under steady-state operation every entering column moves in the same way.)

The inconsistencies in this view of the process can be seen more directly by considering whether material balance is satisfied under the equations developed in the Fitch article.

Before considering the details of this, some comment is needed on the velocity of the constant-concentration characteristic, $v_k$. The evaluation of $v_k$ is not mentioned in the article, but presumably a one-dimensional expression is used. With $Q$ as the total volumetric flow rate, Eq.

5 indicates that $v_k$ is the vertical component of the velocity of a constant-concentration surface relative to the bulk velocity. How is this determined in two-dimensional flow?

Relative to coordinate axes moving with the bulk velocity vector $v_u$, the solids continuity (or material balance) equation for a general point can be written as:

$$\frac{dc}{dt} = -\nabla \cdot (c\mathbf{u}) + \mathbf{u}_p \cdot \nabla c \qquad (1)$$

where $\mathbf{u}$ is the settling velocity of the solids at the point, and $\mathbf{u}_p$ is the velocity of the point itself, both relative to $\mathbf{v}_u$.

Thus, the motion of a constant-concentration point ($dc/dt = 0$) is given by ($\mathbf{u}_p$ becomes $\mathbf{v}_k$):

$$\mathbf{v}_k \cdot \nabla c = \nabla \cdot (c\mathbf{u}) \qquad (2)$$

that is, the velocity of the point is such that its scalar product with the concentration gradient equals the divergence of the solids settling flux.

For one-dimensional thickening this reduces immediately to:

$$v_k = \frac{\partial(cu)}{\partial c} \qquad (3)$$

Thus, under the $u = u^* = u^*(c)$ assumption, a surface of fixed concentration moves with fixed velocity. Presumably, the author takes $v_k$ as evaluated from Eq. 3.

However, in more than one dimension, vectors $\mathbf{v}_k$, $\nabla c$, and $\mathbf{u}$ will generally have different directions, so that there is no simple expression for $\mathbf{v}_k$. Being a scalar equation, Eq. 2 cannot alone yield separate expressions for the vertical and horizontal components of $\mathbf{v}_k$. Even under the $u = u^*$ assumption, the vertical component of $\mathbf{v}_k$ will not generally be constant.

Using $x$ for vertical direction and $y$ for horizontal in two-dimensional rectangular Cartesian coordinates, Eq. 2 with the $u_x = u^*(c)$ assumption and $u_y = 0$ is:

$$v_{kx}\frac{\partial c}{\partial x} + v_{ky}\frac{\partial c}{\partial y} = \frac{d(cu^*)}{dc}\frac{\partial c}{\partial x} \qquad (4)$$

This will only reduce to Eq. 3 if $v_{ky}$ or $\partial c/\partial y$ is zero. Neither will be the case in general; and from the discussion in relation to Figure 3 it appears that the author is assuming neither of these.

So there is a question about the correct evaluation of $v_k$ and its constancy. However, assuming that $v_k$ can be evaluated by the one-dimensional equation, the predictions of the author's analysis can be examined further.

Consider any level in the feed zone (Figure 2). Solids pass through part of the cross-section; and over this part the total solids flow $Q_s$ relative to the equipment must be the same as that at every other level. On the horizontal line in the solids region, the concentration varies according to the analysis presented from the feed concentration $c_0$ at the lefthand wall, to some concentration $c_1$, say, at the boundary with the clear-liquid region. For a given value of $c_1$, the value of $v_k$ ($v_{k1}$ say) is obtained from the slope of the $cu = cu^*(c)$ curve at this concentration and the area $A_{ch}$ occupied by solids at this level from the author's Eq. 5.

To calculate the total solids flow through the solids-occupied area at the level being considered, the variation in concentration and settling velocity along the horizontal line to the wall must be determined. Let the conditions at the righthand end of the solids region, point 1, be used as reference. To avoid unnecessary complications, assume the sediment to be incompressible. Thus, each characteristic starts at the bottom of the column when it is in its leftmost position. The time $t_1$ taken for concentration $c_1$ to move through the column of suspension equals the time taken for the column to move from the wall to position 1 and is given by:

$$t_1 = (y_1 + z_1)/v_{k1}$$

while

$$z_1 = t_1 \cdot v_u$$

since the column is simultaneously moving downward at velocity $v_u$.

Now consider a point at a distance from the wall that is fraction $x$ of the distance to point 1. Since the horizontal flow velocity is taken as uniform, the time $t_x$ taken to reach this point and the distance $z_x$ that the column will have moved down are given by:

$$t_x = x.t_1 = x(y_1 + z_1)/v_{k1} \text{ and } z_x = x.z_1$$

so that

$$v_{kx} = \frac{y_1 + x.z_1}{xt_1} = \frac{v_{k1} - (1-x)v_u}{x} \qquad (5)$$

Then, the concentration at point $x$ is that for which $v_k$ has this value and which will be determined by the slope of the $cu = cu^*(c)$ curve or the slope of the tangent from the initial concentration to the $cu^*$ curve, if the concentration lies in the range covered by this tangent. If there is no concentration with so large a value, however, no characteristic will have reached the point, and so the concentration will be the initial value $c_0$.

Having determined the horizontal profile at the level being considered, the total solids flow rate can be calculated by integration:

$$Q_s = A_{ch}\int_0^1 c_x(u_x^* + v_u).dx$$

If the analysis proposed by the author is correct, then this should be the same as $Q_s$ used to calculate $v_{k1}$, that is, it should be the same as:

$$Q_s = A_{ch}v_{k1}$$

In other words, if the analysis is correct, the following identity should be satisfied:

$$v_{k1} \equiv \int_0^1 c_x(u_x^* + v_u).dx \qquad (6)$$

for any chosen value of $c_1$.

Since $v_{k1}$ depends on a point value of the $cu^* = cu^*(c)$ slope, while the righthand side involves the integral of values calculated via Eq. 5, clearly this will not be satisfied for a general $cu^* = cu^*(c)$ shape.

Given the many inconsistencies in the argument, the author cannot claim to have established that free-settling concentration gradients involving values greater than $c_0$ develop during continuous thickening. The spreading process undergone by the solids after settling from the overflow stream is a complex process, and not amenable to any simple analysis. There is no doubt in my mind that the primary tendency is for thinning rather than thickening, until additional solids retardation (compression) is experienced as the sediment is approached.

Apart from the question of concentration gradient development, the ultimate model produced by the author is one-dimensional, as mentioned above. The settling is considered to occur as though in a column that is uninfluenced by adjacent columns. The fact that the column is considered to be moving horizontally ultimately has no effect on the results; essentially a one-dimensional picture emerges. The only purpose served by the two-dimensional considerations is to assert that, contrary to normal one-dimensional analysis, concentrations greater than $c_0$ spread into the free-settling zone. There does not appear to be any refeience to Eq. 5, for example, later in the article.

As noted at the outset of this discussion, what appears to be the final consequences of the two-dimensional analysis are essentially the same as those arising from Font's one-dimensional analysis. In the latter, the instantaneous sediment height during batch settling and its rate of increase are related to the sediment height in a corresponding continuous thickener (assuming one-dimensional plug-flow in the latter). The correspondence is obtained by equating $dL/dt$ in the batch case with $v_u$ in the continuous case. It is concluded that the sediment height is generally greater in the batch than in the continuous case, as noted by the present author.

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## Reply:

Dixon's criticisms, like those of Concha, are based on misconceptions and misinterpretations, but not on the same ones.

The principal contention in Dixon's letter seems to be that Kynch characteristics will not propagate upward through the suspension as it spreads out in the feed zone. It is asserted that ". . . . the solids stream can only decrease in concentration, approaching $c_i$, during the spreading process." This would be permitted by a one-dimensional mathematical model, but does not represent what would happen in a feed zone, which is, as a matter of empirical fact, two-dimensional. The letter's model of a feed zone (and of several others historically) would be about as shown in Figure 1. In a purely one-dimensional model the feed would have to be introduced uniformly over the entire area, but Dixon's model assumes that it is introduced as an input stream and immediately spreads out over the entire area at feed level. As it spreads out the solids "thin" to concentration $c_i$ and settle through a zone of this concentration to the subjacent compression zone, historically $c_i$ has been termed the "lower conjugate concentration."

### *Fallacy of the lower conjugate concentration zone*

The lower conjugate concentration zone is discussed in considerable detail in Fitch (1972). One-dimensional zone settling or Kynch theory permits only two free settling concentrations to exist at steady state in a thickener. One is that of the critical zone, which will not exist if the thickener is not overloaded. The other is the lower conjugate concentration, which has been shown to be unstable.

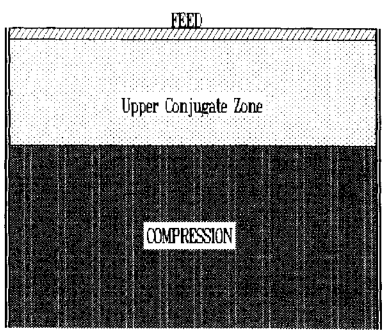


**Figure 1. Physical interpretation of Dixon mathematical model.**

One-dimensional continuity would demand that the lower conjugate concentration exist between the feed point and whatever higher concentrations exist below. But there is no necessity to assume one-dimensional continuity. And many thickeners show a mud line far below the feed point with clear supernatant above, at least near the periphery where samples are commonly provided, so a lower conjugate concentration empirically does not have to exist.

If feed is more concentrated than a lower conjugate concentration, there is a large body of literature in the civil engineering field, which recognizes that it will plunge to the bottom of such a zone like a submerged waterfall as a density current (for example, Sawyer, 1956; Fitch and Lutz, 1960).

For a more complete discussion of the matter refer to the cited articles. The conclusion is that the flow pattern shown in Figure 1 cannot exist at steady state in a thickener. You cannot float a zone of higher concentration over one of lower concentration. Such a pattern is hydrodynamically unstable. Thus, the "thinned" zone of concentration $c_i$ entailed by Dixon's model cannot exist in a real thickener. As noted in the introduction to the subject article, "New feed suspension, which contains settleable solids and therefore is denser than the supernatant, plunges as a submerged waterfall to its level of hydrostatic equilibrium. There it spreads out horizontally as a density current, either just above the compression zone or above a critical zone if one exists."

### *Kynch characteristics*

If, as required for hydrodynamic stability, the feed plunges to its level of hydrostatic equilibrium and there spreads out, it will be bounded below by a zone of higher concentration, normally a compression zone. Thus, there is a boundary or front between the feed current and the subjacent compression zone. In the boundary layer, all concentrations between that at the bottom of the feed layer and that in the subjacent zone will be present. If the concentrations are such that Kynch characteristics can arise, they will do so and propagate upward through the feed layer as it spreads out. There is no way the feed layer could thin out by collapsing into the compression zone.

The velocity at which a characteristic would move through the feed layer would be the vector sum of two components: the velocity $v_k$ at which the characteristic propagated with respect to the suspension in its neighborhood; and the transport velocity of the neighborhood. The propagation velocity of a Kynch characteristic, like any velocity, is a vector, but in the absence of lateral accelerations is always vertical and is constant by definition for any given characteristic. It thus has no horizontal components and is independent of the transport velocity. It will be the same in a two-dimensional feed zone as it would be in a one-dimensional batch test. For example, in a batch test, the characteristic would propagate at the same velocity relative to the column even if the test were being carried out in a railway car traveling at any constant lateral velocity. $v_K$ is not, as assumed by Dixon, a velocity relative to $v_u$, but relative to the local transport velocity, whatever it may be at the point. In the cross-flow model it will be relative to a velocity whose $x$ component is $v_u$. The transport velocity, on the other hand, will be a function of the flow patterns. In a one-dimensional model it will be vertical and equal to $v_u$. In the hypothetical cross-flow model it will have an $x$ component equal to $v_u$ and a $y$ component that is also constant. In a real thickener it will vary widely from point to point.

In a real thickener the feed layer would have a complex flow pattern and shape. It would have to satisfy both hydrodynamic and particle dynamic constraints. Calculating the flow patterns and the concentration existing at any point would therefore be extremely complicated. However, for thickener design purposes it would be unnecessary and pointless. Because of the area principle, the *area demands* for the feed zone will be independent of the flow pattern. The depth of the density current will not be, but is not a significant factor in practical design.

### *The cross-flow model*

While we cannot practically determine the actual flow patterns in the feed zone, we can assume a hypothetical flow pattern and solve for its area demands. Because of the area principle, the area demands thus calculated will be also valid for the actual flow pattern, whatever it may be. Our aim in this article, as in nearly all of my other articles, is to improve and facilitate thickener design procedures. We would like to relate the area demands of a real thickener to results from a batch settling test. Therefore, we arbitrarily choose a flow pattern for our hypothetical feed zone that will appear and can be treated as one-dimensional in Lagrangean coordinates. As shown in the article, in such a coordinate system the two-dimensional behavior resembles batch settling closely (but not precisely). For determining area demands it can be treated as one-dimensional. That is what is done in the cross-flow model described.

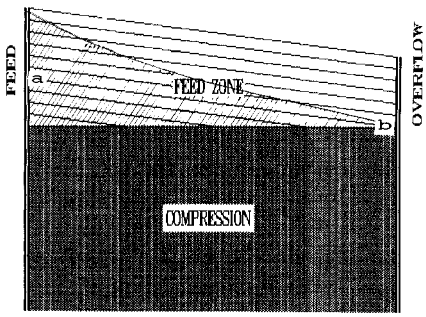


**Figure 2. Flow pattern in cross-flow model.**

It should be fully realized that the cross-flow model does not purport to be a possible one for the actual flow patterns in a thickener. It is, for one thing, hydrodynamically unstable completely. As Dixon notes, "it involves discontinuities at the side boundaries that cannot be avoided by any reasonable assumption," but the area principle is not restricted to real models or to ones that are hydrodynamically sound. The hypothetical flow patterns need only to satisfy material balance. Hydraulically the cross-flow model bears little resemblance to the actual flow patterns in the thickener, and what limited relevance this completely hypothetical model has to the real flow pattern wold not be apparent without first establishing the area principle.

In the cross-flow model, by definition, all flow vectors in the feed zone are equal. As recognized by Concha, this entails that the feed is introduced uniformly over the entire depth of the feed zone, ". . . . the feed enters homogeneously at $y = 0$, $0 <= x <= L$." The feed could not possibly be restricted to $y=0$, $x=0$, as "a point source at the top left corner" as incorrectly inferred in Dixon's letter. For those who fail to perceive this, Figure 3 of the subject article might be misleading. Figure 2 of this response is, therefore, given to explicate more clearly the assumed flow patterns.

Above the boundary between feed and compression zones the flow lines slant downward to the right, in accord with the assumptions of the model. Below the boundary they are vertical, as required by the one-dimensional flow assumed there. Flow lines reaching the boundary in the feed zone lose their $y$ component of velocity, but retain the $x$ component. Flow, including solids and liquid, is taken uniformly from the bottom of the feed zone, and this flow past the boundary is necessary for material balance. If it did not exist, the downward flow postulated in the compression zone would have no source.

Dixon's "lowest overflow streamline" is marked a--b in this response. It does not, as in Dixon's interpretation, ". . . . pass between feed and overflow points close to the top surface." First, neither feed nor overflow occurs at a point in the cross-flow model, although it would do so in a real thickener. Second, the lowest overflow streamline is at no point anywhere near the top surface. Its origin is just above flow lines destined for the underflow, and its termination is at the bottom of the feed zone. Further, it is not true, as Dixon states, that "above the lowest overflow streamline, the liquid flow is predominantly horizontal, . . . . while below the lowest overflow streamline it is predominantly vertical." The streamlines below the lowest overflow streamline have the same slope in the feed zone as they do above it. The concentration of the solids crossing this flow line at any point is that of the Kynch characteristic surfacing at that point and thus is highly variable. Since all we hope to discover from the cross-flow model is the maximum possible solids flux, all we are interested in is the fully-loaded condition. Under this condition, the last of the solids crosses this lowest overflow streamline just as it reaches the effluent side of the hypothetical basin. All the solids will then have crossed the line, and clarification requirements are fully satisfied.

Dixon asserts, "however, the author also assumes a uniform downward velocity $v_u$ throughout the feed zone, which suggests flow below the lowest overflow streamline. The two are incompatible." Material balance requires that if there is a downward velocity component $v_u$ in the feed zone, then in a thickener of constant area this velocity component must exist at all levels, including those below any streamline such as the lowest overflow one. Thus, the flow pattern postulated by Dixon does not satisfy steady-state material balance.

The misinterpretation of the flow pattern actually presented in the article is further evidenced by the following statement: "The envisioned passage of effectively batch settling columns from one side to the other means that all the underflow liquid and solids are deposited against the far side of the thickener." Under the flow pattern postulated in the article and shown in Figure 2 of this response, all the underflow liquid and solids are transferred into the compression zone uniformly as the settling columns pass across the basin. Assuming that the thickener is not underdesigned, all that is left in the feed zone at the discharge end is supernatant. This is also clearly shown in Figure 3 of the article.

The calculations in the letter purporting to show inconsistencies in the article are based on a misconception. The letter asserts, "solids pass through part of the cross-section, and over this part the total solids flow $Q_s$ relative to the equipment must be the same as at every other level." This is the misconception. The solids are originally distributed uniformly from top to bottom of the feed zone at the lefthand or influent side. The solids originally below any given level in the equipment do not have to settle through it. Therefore, the total solids flow past one level in the feed zone will not be the same as that at another level.

Dixon has resurrected the one-dimensional concept of a lower conjugate concentration. Because he has no doubt as to its validity, he uses it to challenge that Kynch characteristics can propagate upward through the density current in the feed zone. Although the lower concentration zone model can satisfy material balance, it is shown theoretically that it would be unstable hydrodynamically and is contraindicated empirically. It cannot be cited validly as a counterexample. So, "given the many inconsistencies in the argument," the letter writer "cannot claim to have established that free-settling concentration gradients involving values greater that $c_o$" do *not* develop during continuous thickening." The criticisms embodied in the letter derive from misconceptions and/or misinterpretations, and are not relevant to what was actually printed. Like those of Concha, they do not disclose any errors in the article.

It is hoped that this response will aid those who have difficulty following the logical arguments in the article.

(*Note: For the notation and literature cited, see Fitch's response to Concha's letter.*)

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